

COMPLEX NUMBERS REFERENCE SHEET

FOUNDATIONS

Imaginary Unit $i = \sqrt{-1}$, $i^2 = -1$

Standard Form $z = a + bi$

Conjugate $\bar{z} = a - bi$

Modulus (Magnitude) $|z| = \sqrt{a^2 + b^2}$

POLAR & EXPONENTIAL

Polar Form $z = r(\cos \theta + i \sin \theta)$

Euler's Formula $e^{i\theta} = \cos \theta + i \sin \theta$

Exponential Form $z = re^{i\theta}$

Argument $\theta = \arg(z) = \tan^{-1}(b/a)$

OPERATIONS (POLAR)

Multiplication $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

Division $z_1 / z_2 = (r_1 / r_2) e^{i(\theta_1 - \theta_2)}$

Powers (De Moivre) $z^n = r^n (\cos n\theta + i \sin n\theta)$

ROOTS

n-th Roots $w_k = r^{1/n} e^{i(\theta + 2k\pi)/n}$

Range $k = 0, 1, 2, \dots, n-1$

LOGARITHMS & HYPERBOLICS

Complex Logarithm $\ln(z) = \ln|z| + i(\arg z + 2k\pi)$

Principal Value $\text{Ln}(z) = \ln|z| + i\text{Arg}(z)$

Relation to Hyperbolics $\sin(iz) = i \sinh(z)$; $\cos(iz) = \cosh(z)$

Note: θ is expressed in radians. Principal Argument $\text{Arg}(z) \in (-\pi, \pi]$.