

COMPLEX NUMBERS: EXPONENTIAL FORM REFERENCE

$$z = r(\cos \theta + i \sin \theta) = re^{i\theta}$$

$$\text{Where } r = |z| = \sqrt{a^2 + b^2} \text{ and } \theta = \arg(z)$$

Rectangular (a + bi)	Polar [r, θ]	Exponential ($re^{i\theta}$)
1	[1, 0]	$1e^{i0}$
i	[1, $\pi/2$]	$1e^{i\pi/2}$
-1	[1, π]	$1e^{i\pi}$
-i	[1, $3\pi/2$]	$1e^{i3\pi/2}$
$1 + i$	[$\sqrt{2}$, $\pi/4$]	$\sqrt{2}e^{i\pi/4}$
$\sqrt{3} + i$	[2, $\pi/6$]	$2e^{i\pi/6}$

Operations

- **Multiplication:** $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$
- **Division:** $z_1 / z_2 = (r_1 / r_2) e^{i(\theta_1 - \theta_2)}$
- **Powers:** $z^n = r^n e^{in\theta}$

Key Identities

- $e^{i\pi} + 1 = 0$
- $\sin \theta = (e^{i\theta} - e^{-i\theta}) / 2i$
- $\cos \theta = (e^{i\theta} + e^{-i\theta}) / 2$