

# COMPLEX NUMBERS & PLANE GEOMETRY

## FUNDAMENTAL FORMS

Rectangular:  $z = x + iy$

Polar:  $z = r(\cos \hat{\theta} + i \sin \hat{\theta})$

Exponential:  $z = re^{i\hat{\theta}}$

## VECTOR REPRESENTATION

Real (Re)

Imaginary (Im)

## GEOMETRIC OPERATIONS

### Operation

### Geometric Effect

Addition ( $z + w$ )

Translation by vector  $w$

Multiplication ( $z \hat{\cdot} e^{i\hat{\theta}}$ )

Rotation by angle  $\hat{\theta}$

Multiplication ( $z \hat{\cdot} k$ )

Scaling (Dilation) by factor  $k$

Conjugation ( $z^*$ )

Reflection across Real axis

## DISTANCE & CIRCLES

Distance:  $|z_1 - z_2|$

Circle eq:  $|z - z_0| = r$

Line eq:  $|z - a| = |z - b|$

\*Where  $z_0$  is the center and  $r$  is the radius. The line equation represents the perpendicular bisector of segment  $ab$ .

## DE MOIVRE'S THEOREM

$$z^n = r^n [\cos(n\hat{\theta}) + i \sin(n\hat{\theta})]$$

# ROOTS OF UNITY

$$\omega^k = e^{i(2\pi k/n)}$$

for  $k = 0, 1, \dots, n-1$

Reference Material  $\hat{=}$  Complex Analysis & Euclidean Geometry Integration