

IMAGINARY NUMBERS & COMPLEX ANALYSIS

Calculus Student Reference Sheet

THE IMAGINARY UNIT

i^1	i	i^{-1}
i^2	-1	
i^3	$-i$	$-i^{-1}$
i^4	1	(Full Cycle)

Pattern repeats: $i^n = i^{n \bmod 4}$

COMPLEX FORM

$$z = a + bi$$

- **Re(z):** Real part (a)
- **Im(z):** Imaginary part (b)
- **Conjugate (\bar{z}):** $a - bi$
- **Magnitude $|z|$:** $\sqrt{a^2 + b^2}$

EULER'S FORMULA & POLAR FORM

$$e^{i\theta} = \cos(\theta) + i \sin(\theta)$$

Polar Form:

$$z = r(\cos \theta + i \sin \theta)$$

Exponential Form:

$$z = re^{i\theta}$$

DE MOIVRE'S THEOREM

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

CALCULUS IDENTITIES

Derivative

$$d/dz [e^{az}] = ae^{az}$$

Definition of Sine

$$\sin(z) = (e^{iz} - e^{-iz}) / 2i$$

Definition of Cosine

$$\cos(z) = (e^{iz} + e^{-iz}) / 2$$

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